



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Lecture 9: Basic Optics for Optical Sensing-VII

Absorption and dispersion: Drude model for conductors, Bulk Plasmons

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Absorption

When the refractive index is complex: $n = n_r \pm i\kappa$

κ -The extinction coefficient !. The sign depends on the choice: $e^{\pm ikz}$

The relation to real and imaginary parts of the dielectric function:

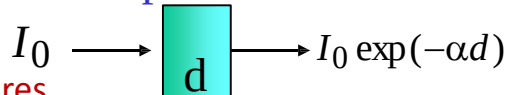
$$\epsilon_r = n_r^2 - \kappa^2 \text{ and } \epsilon_i = 2n\kappa$$

A plane wave will be decaying as: $E(z) = \exp(ik_0 n_r z) \exp(-k_0 \kappa z)$

The intensity is decaying as: $|E|^2 = \exp(-2k_0 \kappa z)$

The absorption coefficient is defined as:

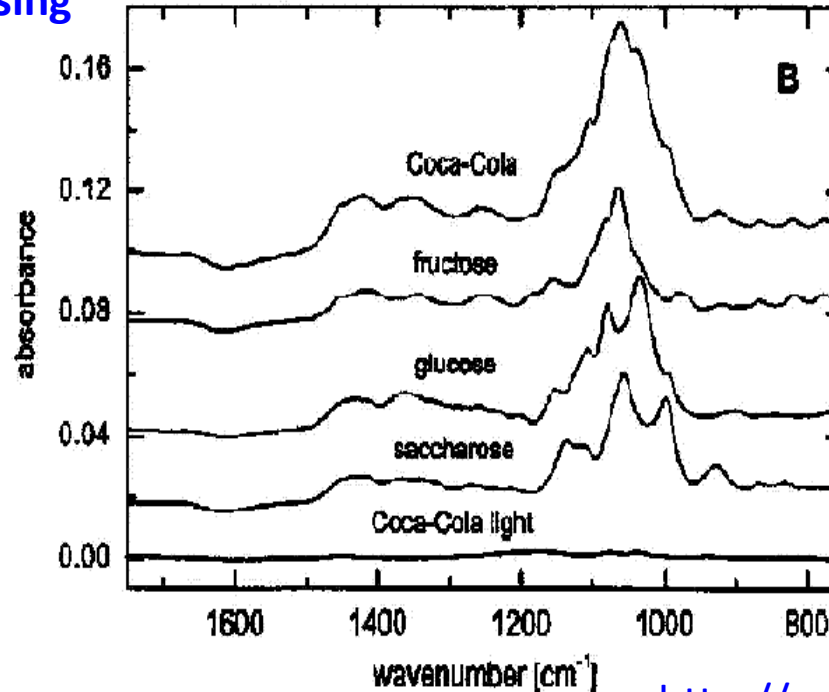
$$\alpha = 2k_0 \kappa = \frac{4\pi\kappa}{\lambda_0}$$



If we ignore reflections at the boundaries!

Evanescent Wave Absorption Sensors using Silver Halide IR Fibers

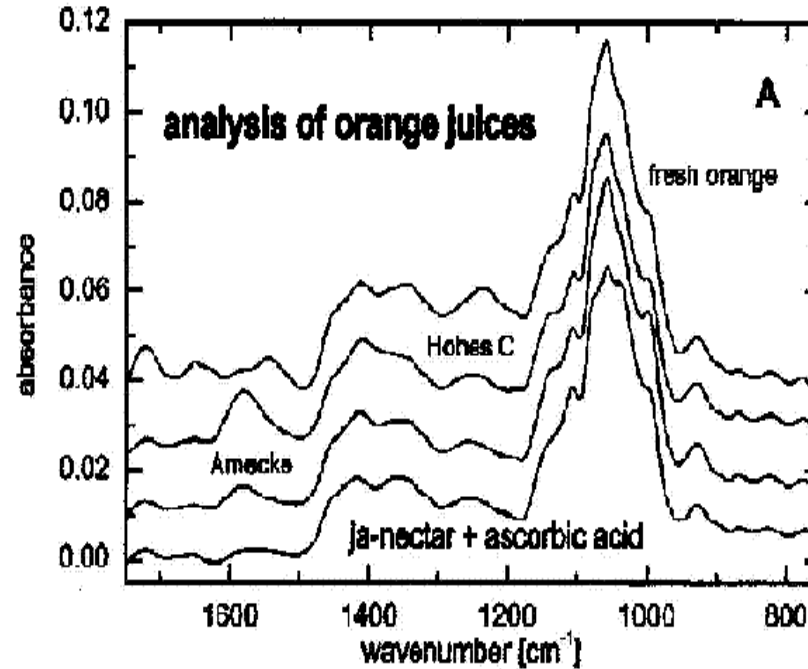
Glucose Sensing



<http://www.forc.gpi.ru/lab/ir/Primenen2.html>

Evanescent Wave Absorption Sensors using Silver Halide IR Fibers- II

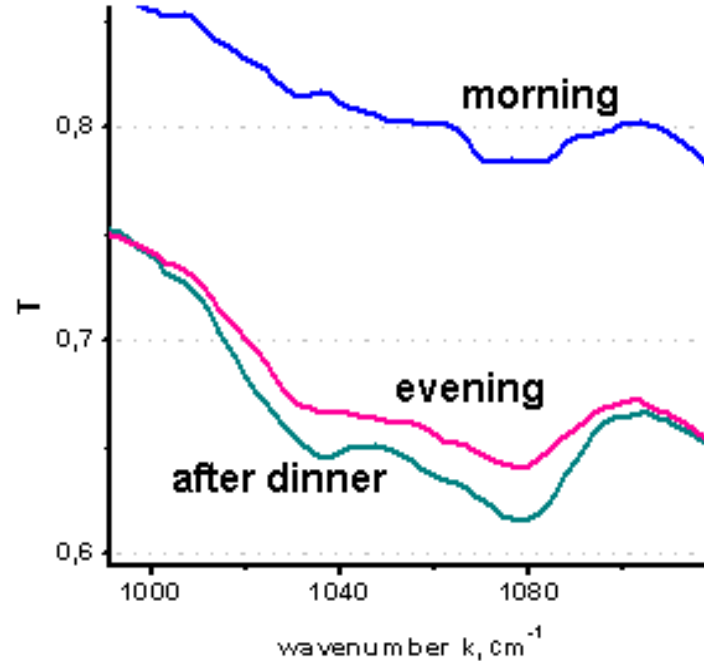
Orange Juice Analysis



I. Abdulhalim, BOSMI Lectures

Evanescent Wave Absorption Sensors using Silver Halide IR Fibers-III

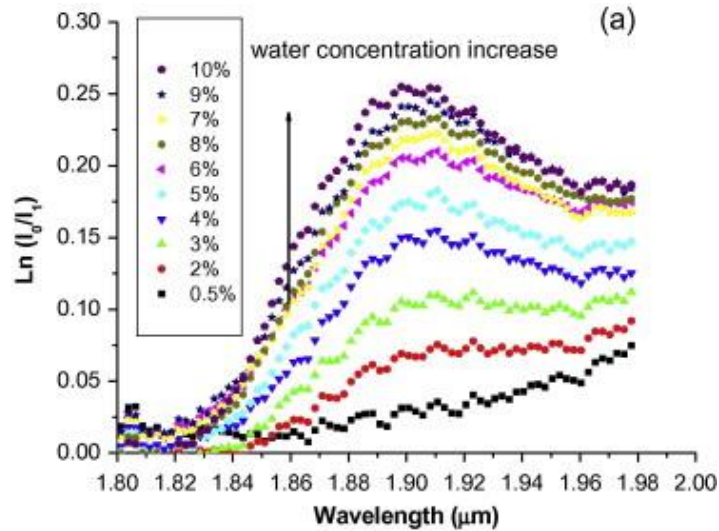
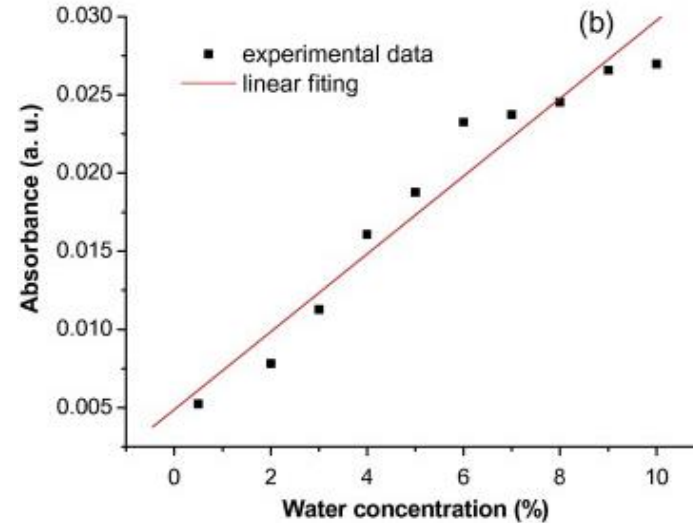
**Skin spectrum variation
due to glucose
concentration variation**



I. Abdulhalim, BOSMI Lectures

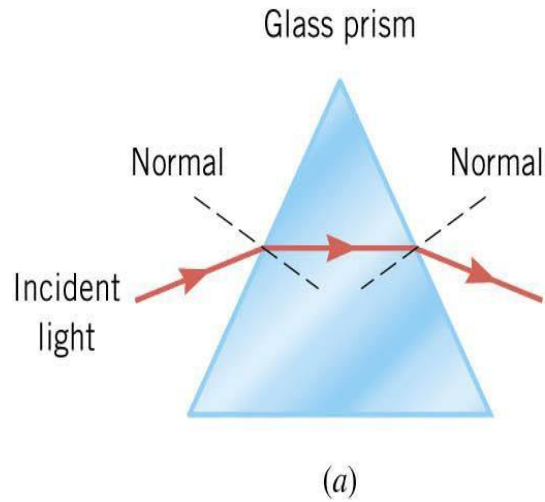
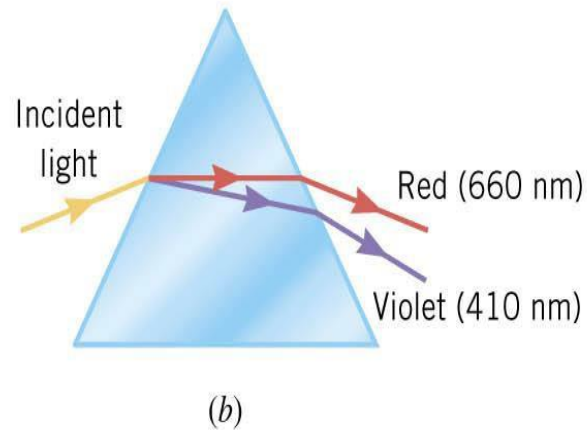


Evanescent wave Absorption based Detection of Ethanol in Water

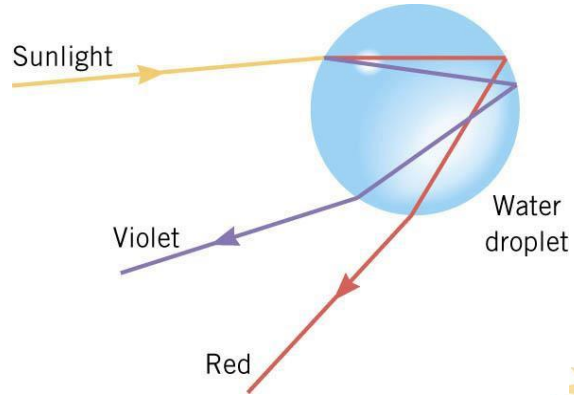
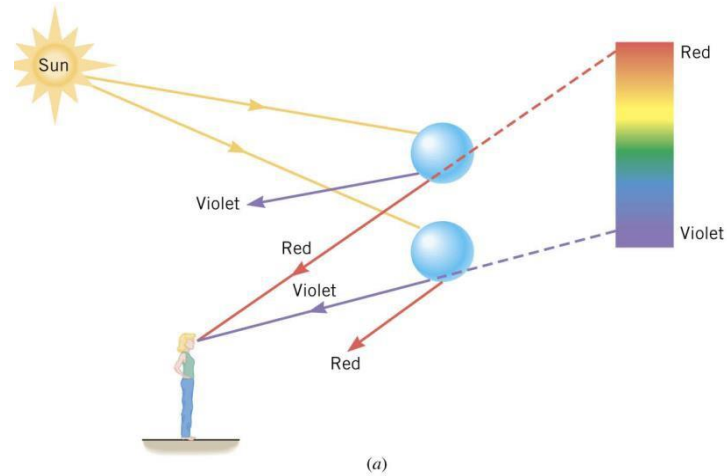


F.B. Xiong, D. Sisler, Opt. Commun. 283, 1326–1330 (2010)

Dispersion - Prism



Dispersion - Rainbow



Dispersion – Lorentz Model

Oscillating bound electron model (Lorentz model):

$$m_e(r(t) + \gamma r(t) + \omega_0^2 r(t)) = -q_e E_{eff}(t)$$

inertia

damping

restoring
force

applied
force

Now use:

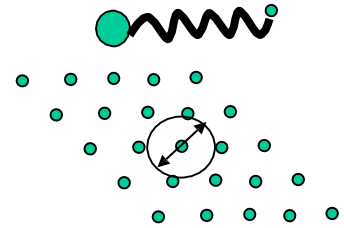
$$E_{eff}(r, t) \approx E(r, t) + \frac{4\pi P(r, t)}{3}$$

and

$$P(r, t) = N\mu_a = -Nq_e r(t)$$

Mossotti (1850) equation (CGS)

Depolarization field



Dispersion – cont'

Then:

$$\ddot{P}(t) + \gamma\dot{P}(t) + (\omega_0^2 - \frac{4\pi Nq_e^2}{3m_e})P = \frac{Nq_e^2}{m_e} E(t)$$

Use: $P(t) \propto \exp(i\omega t)$ and: $D = \epsilon E = E + 4\pi P$ (CGS)

To get:

$$\epsilon(\omega) \approx 1 + \frac{b^2}{\omega_{0s}^2 - \omega^2 - i\gamma\omega}$$

Where:

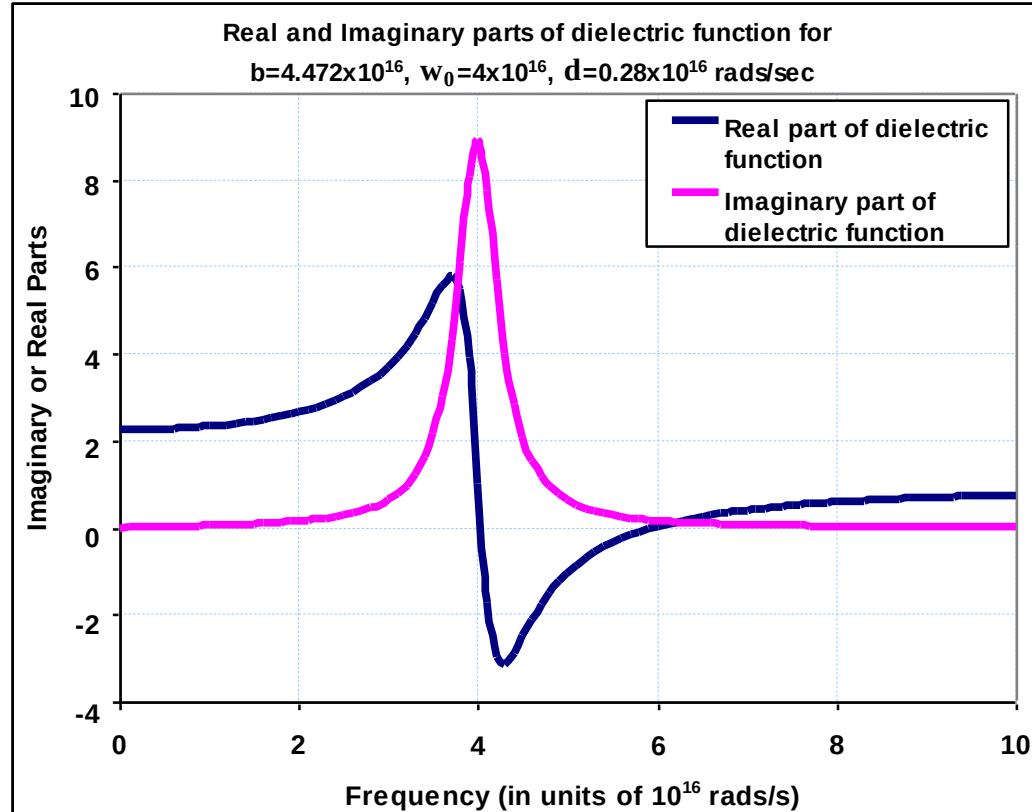
$$\omega_{0s} = \sqrt{\omega_0^2 - \omega_c^2}$$



Resonance frequency is shifted by frequency of oscillating dipoles

$$\omega_c^2 = b^2/3 = 4\pi Nq_e^2/3m_e$$

Dispersion – cont'



RI Dispersion in Glasses

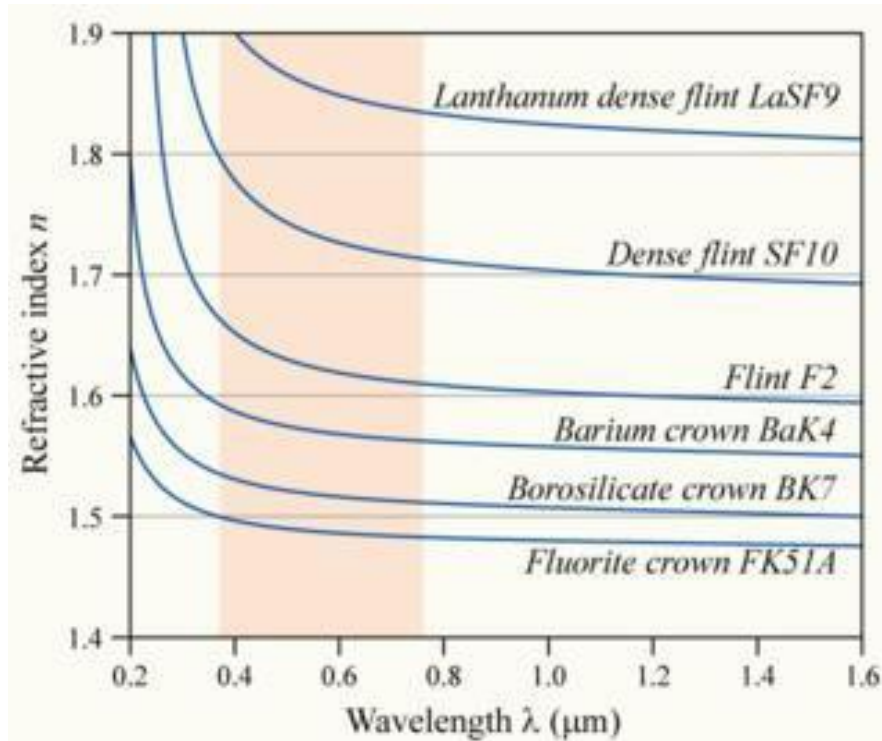
Glasses are usually transparent in order to be used for imaging, hence their refractive indices are real and follow one of the following formulae:

Cauchy:
$$n(\lambda) = n_{\infty} + \frac{a}{\lambda^2} + \frac{b}{\lambda^4} + \dots$$

Sellmeier:
$$n(\lambda) = \sqrt{1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} + \dots}$$

$$n(\lambda) = \sqrt{A + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} + \dots}$$

RI Dispersion in Glasses II



For BK7:

Coefficient	Value
B_1	1.03961212
B_2	0.231792344
B_3	1.01046945
C_1	$6.00069867 \times 10^{-3} \mu\text{m}^2$
C_2	$2.00179144 \times 10^{-2} \mu\text{m}^2$
C_3	$1.03560653 \times 10^2 \mu\text{m}^2$

See one of the useful websites:

<http://refractiveindex.info/>

Dispersion of Free Carriers

In space with free charge carriers such as free electrons in metals or ions in the ionosphere no restoring force and no damping hence the equation we derived for the oscillating dipole becomes:

$$\ddot{P}(t) = \frac{Nq_e^2}{m_e} E(t) \quad \Rightarrow \quad P(t) = \frac{-Nq_e^2}{m_e \omega^2} E(t)$$

Use:

$$D = \epsilon E = E + 4\pi P \quad \Rightarrow \quad \epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

Where the plasma frequency:

$$\omega_p^2 = 4\pi Nq_e^2 / m_e = b^2 \quad (\text{CGS})$$

$$\omega_p^2 = Nq_e^2 / \epsilon_0 m_e \quad (\text{SI})$$

Dispersion of free Carriers – cont'

From the relation for bound electrons:

$$\varepsilon(\omega) \approx 1 + \frac{b^2}{\omega_{0s}^2 - \omega^2 - i\gamma\omega}$$

Substitute $\omega_{0s} = 0$ for free carriers to get:

$$\varepsilon(\omega) \approx 1 - \frac{b^2}{\omega(\omega + i\gamma)} = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \xrightarrow{\gamma=0} 1 - \frac{\omega_p^2}{\omega^2}$$

↑
Drude Term

Which is the same function obtained without damping

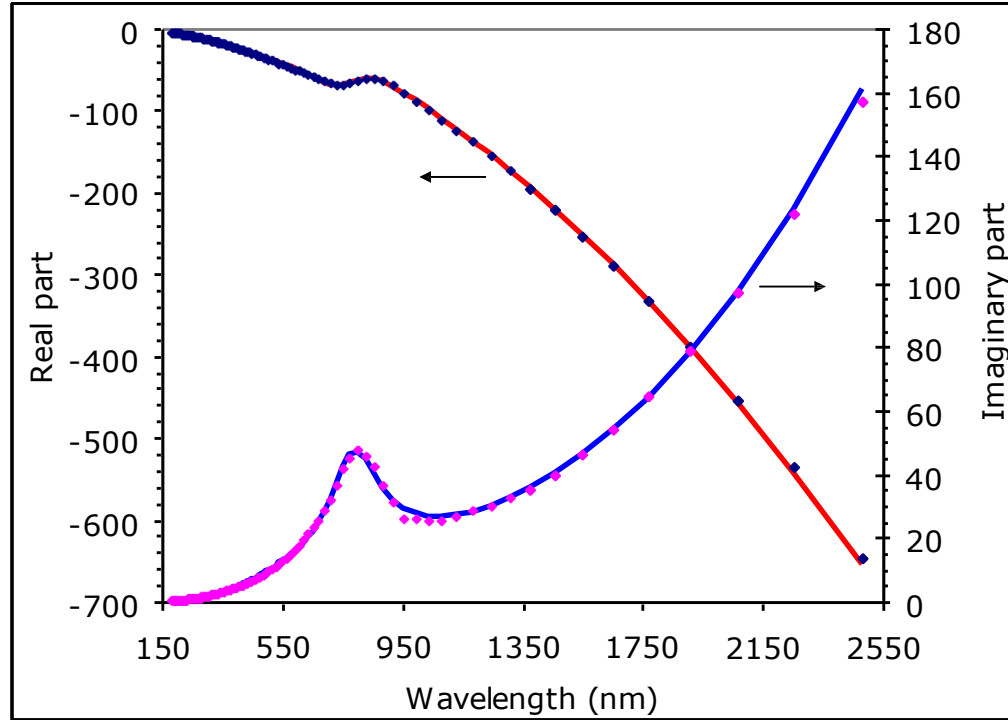
Dispersion of Metals

Better described by a combination of Drude term and two oscillators:

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega^2 + i\Gamma_p\omega} + \sum_j \frac{f_j\omega_j^2}{\omega_j^2 - \omega^2 + i\Gamma_j\omega}$$

Metal	ε_{∞}	$\omega_p(\text{eV})$	$\Gamma_p(\text{eV})$	f_1	$\omega_1(\text{eV})$	$\Gamma_1(\text{eV})$	f_2	$\omega_2(\text{eV})$	$\Gamma_2(\text{eV})$
Ag	2.302	8.807	0.082	0.439	4.602	0.761	1.098	6.008	2.393
Au	3.952	8.542	0.070	-85.116	2.6	3.712	90.046	2.789	4.27
Al	0.952	13.335	0.117	8.119	1.579	0.407	9.908	2.006	1.942

Fit for Aluminum



Measurements are based on spectroscopic ellipsometry of Sopra (<http://www.sopra-sa.com/>)

Summary

1. Evanescent wave absorption-based sensors were discussed.
2. Dispersion characteristics using Lorentz damped oscillator model were discussed and Drude model was achieved.

Thank You